

Chapter 2

Non Linear Techniques for Studying Complex Systems

*Clouds are not spheres, mountains are not cones,
coastlines are not circles,
and bark is not smooth,
nor does lightning travel in a straight line*

—Benoit Mandelbrot

2.1 Introduction

In the words of Mendelbrot, a fractal is defined as a “A rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced/size copy of the whole.” In the last two or three decades much research has gone into the research of chaos theory which mainly deals with the dynamics of different complex systems that are found in nature. Chaos can be found everywhere, from nature’s most intimate considerations to art of any kind. Chaos theory is the study of how systems that follow simple, straightforward, deterministic laws can exhibit very complicated and seemingly random long term behavior. With the advent of chaos theory we now have a number of robust tools which can deal with a variety of complex signals like DNA sequence, earthquake, turbulent water flows, weather changes, financial time series, faults in bone and most importantly human bio-signals. The use of fractal dimension has opened a whole new plethora of studies dealing with complex dynamics of these signals. This chapter is essentially a detailed description of the different algorithms used in various sections of this book. We start off with conventional Fourier decomposition methods which were used to compare with the non-linear methods and then continue with the various non-linear methods which have been used for assessment of various EEG and music signal data.

Most frequently used techniques of EEG analysis such as Fourier decomposition are essentially linear, but the human brain is the most complex nonlinear system. The scalp EEG arises from a large number of neurons, whose interactions with the neighboring neurons as well as with remote neurons are ought to be nonlinear and thus they can generate fluctuations that are not best described by linear

decomposition. Hence, it is important to analyze these signals with the help of techniques which are robust against non-stationarities inherent in these signals as well as the spikes present. Classical methods of signal analysis work well mostly on stationary signals, so we need a new solution—new methods. Non-linear dynamical analysis has emerged as a novel method for the study of complex systems in the past few decades. The non-linear analysis method is effectively applied to electroencephalogram (EEG) data to study the dynamics of the complex underlying behavior. The growth of this method as a tool for mental health evaluation mainly rests on the non-invasive nature of EEG. The approach is based on the principles of non-linear dynamics and deterministic chaos that involves the characterization of the system attractors with its invariant parameters.

For a neuronal network such as the brain, nonlinearity is introduced even on the cellular level, since the dynamical behavior of individual neurons is governed by threshold and saturation phenomena. Moreover, the hypothesis of an entirely stochastic brain can be rejected due to its ability to perform sophisticated cognitive tasks. For these reasons, the electroencephalogram (EEG) appears to be an appropriate area for nonlinear time series analysis techniques, the practical spin-off from the theory of deterministic chaos.

Nonlinear dynamics (more precisely in this case—chaos theory) provides many new ways of analyzing signals, such as fractal methods. Some of these methods determine the scaling exponent of the signal which indicates the presence or absence of fractal properties (self-similarity). The FD of a waveform represents a powerful tool for transient detection. This feature has been used in the analysis of ECG and EEG to identify and distinguish specific states of physiological function. The fractal tool thus can essentially be compared with a mathematical microscope zooming its way into the inherent complex patterns of the signal and deducing a complex scaling exponent from the apparent random pattern. Many robust algorithms are available to determine the FD of the waveform. There are different methodological approaches and their respective statistical parameters to capture fractality namely Correlation dimension, Lyapunov exponent, Box counting method etc. These are very sensitive to noise and require the stationary condition while EEG signals are highly non stationary. For this reasons, we use a nonlinear method named Detrended Fluctuation Analysis (DFA) followed by Multifractal Detrended Fluctuation Analysis (MFDFA) which has the ability to capture scale varying nature of different naturally occurring time-series signals. To further elucidate how the internal dynamics of one signal affects the other, or in other words what is the degree of cross-correlation among the two, we take the help of Multifractal Detrended Cross Correlation analysis (MFDXA) which gives the cross-correlation coefficient as an output. Other unique and new methods used for feature extraction from EEG signals like neural jitter/neural shimmer and pitch extraction have also been discussed in detail in this chapter.

Music signals are far more complex in their dynamics as compared to the EEG signals due to the superposition of a large number of frequency components and hence their treatment with conventional power spectrum techniques is not justifiable. The MFDFA technique is much more accurate than the conventional DFA

technique especially in case of music signals because of the fact that there are segments with extremely large variation as well as segments with very small variation (i.e. they are multifractals) therefore, the normal distribution considering second order RMS variation cannot be applied and all the q-order moments need to be considered. This method is very useful in the analysis of various non-stationary time series and it also gives information regarding the multifractal scaling behaviour of non-stationary signals. The music signals were initially processed with music analysis software Wavesurfer (Sjölander and Beskow 2000) and Cool Edit (Johnston 1999) before being used for listening test and EEG data acquisition.

The raw EEG signal is generally contaminated by various types of external artifacts such as eye blinks, muscular movement etc. Eye blinks and eye movements are characterized by frequency of less than 4 Hz and high amplitude. So, it is essential to get an EEG data which are free from these artifacts, which may induce considerable error in the final results. We propose a novel data-driven noise removal technique called Empirical Mode Decomposition (EMD) which helps in generation of noise/artifact-free EEG data in a few steps.

The Wavelet Transform Technique (WT) have also been utilized as a superior alternative to conventional Fourier Transform (FT) technique which decomposes the complete EEG signal into its five characteristic frequency bands viz. Delta (δ): 0–4 Hz (2) Theta (θ): 4–8 Hz (3) Alpha (α) 8–13 Hz (4) Beta (β): 13–30 Hz (5) Gamma (γ): 30–50 Hz. As most of the previous works in the domain of EEG signal processing uses the spectral power from these bands as a feature for distinguishing one brain state from another, we felt the need to compare the results obtained from non-linear analysis of these very spectral bands. Hence the WT technique has been applied in a number of studies elaborated later in this book. In the next sections, we present algorithms of the various techniques utilized later in this book, some of which have been used widely while some are entirely new based on our research in this subject. Firstly, the novel EMD technique utilized for removing the artifacts from raw EEG signal.

2.2 Empirical Mode Decomposition (EMD)

EMD is a decomposition method for non-stationary and nonlinear signals (Huang et al. 1998). The EMD technique decomposes a signal into a number of intrinsic mode functions (IMFs) that represent fast to slow oscillations. An IMF is a function that satisfies two conditions:

(1) the number of extrema and the number of zero crossings must either be equal or differ by at most one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. To obtain an IMF from the original signal x , a sifting process is performed (Huang et al. 1998) as follows:

First, all extrema of the original signal x need to be identified. All local maximum points are connected by a cubic spline line to form the upper envelope e_u . All

local minima points are connected likewise to form the lower envelope e_l . The mean of e_u and e_l , a_1 , is calculated as:

$$a_1 = \frac{(e_u + e_l)}{2} \quad (2.1)$$

The difference between the original signal and the mean is defined as the first component h_1 :

$$h_1 = x - a_1 \quad (2.2)$$

In the next sifting process, h_1 is treated as the signal, and the mean a_{11} of its local maxima and local minima is found. Thus, we have:

$$h_{11} = h_1 - a_{11} \quad (2.3)$$

Subsequently, we can repeat this sifting procedure k times until h_{1k} is an IMF, with:

$$h_{1k} = h_{1(k-1)} - a_{1k} \quad (2.4)$$

Therefore, the first IMF component derived from the original signal is designated as:

$$c_1 = h_{1k} \quad (2.5)$$

The sifting process has been stopped when an IMF has been established by limiting the size of the standard deviation (SD), calculated from the two consecutive sifting sequences as below:

$$SD = \sum_{t=0}^T \frac{[h_{1(k-1)}(t) - h_{1k}(t)]^2}{h_{1(k-1)}^2(t)} \quad (2.6)$$

A typical value for SD can be set between 0.2 and 0.3 (Huang et al. 1998). In our case the value was set to 0.25. To extract the 2nd IMF component, we remove c_1 from the original signal x :

$$r_1 = x - c_1 \quad (2.7)$$

The residual r_1 is treated as a new signal, and the same sifting process is applied to obtain the 2nd IMF component c_2 and the residual:

$$r_2 = r_1 - c_2 \quad (2.8)$$

This procedure is repeated on the subsequent residuals r_j 's, until the final residual r_J no longer contains any oscillation information,

$$r_j = r_{j-1} - c_j \quad (2.9)$$

By summing up Eqs. (2.7)–(2.9), we can obtain:

$$x = \sum_{j=0}^J c_j + r_J. \quad (2.10)$$

Thus, original signal x is decomposed into J empirical modes c_j 's and a residue r_J .

Since, the artifacts lies in the low frequency regions (<3.5 Hz) (Bizopoulos et al. 2013; Jung and Saikiran 2016), the IMFs that appear in this band are rejected. Thus, the filtered signal is the sum of the remaining IMFs and more specifically, only the first few IMFs including the residue were kept (Bizopoulos et al. 2013). We have obtained noise free EEG data for all the electrodes using the EMD technique and used this data for further analysis and classification of acoustic stimuli induced EEG features.

Figure 2.1a–k shows a representative figure of the F3 electrode in 10 s duration which was subjected to EMD technique and the noise-free EEG data. The sifting process was continued until the final residue is a constant, a monotonic function, i.e. a function with only one maxima or minima from which no more IMF's can be derived. We have set the value of SD to be 0.25 after which the sifting process has been stopped.

The noise-free signal obtained after the removal of muscular and blink artifacts has been used as the input for the wavelet transformation technique (Fig. 2.2).

2.3 Wavelet Transform

Wavelet transform forms a general mathematical tool for signal processing with many applications in EEG data analysis (Selesnick et al. 2005; Dimoulas et al. 2007; Hazarika et al. 1997). Its basic use includes time-scale signal analysis and decomposition of EEG signal. We have used WT technique to decompose the noise cleaned EEG signal obtained from the previous step into various frequency bands i.e. alpha (9–13 Hz), delta (1–3 Hz), theta (4–8 Hz), beta (14–30 Hz). The DWT (Akin et al. 2001) analyzes the signal at different frequency bands with different resolutions by decomposing the signal into a coarse approximation and obtains detailed information. DWT generally employs two sets of functions, called the scaling functions and wavelet functions, associated with low pass and high pass filters, respectively. The decomposition of the noise free signal into different frequency bands is done by successive high pass and low pass filtering of the time domain signal. The original signal $x[n]$ is first passed through a half band high pass

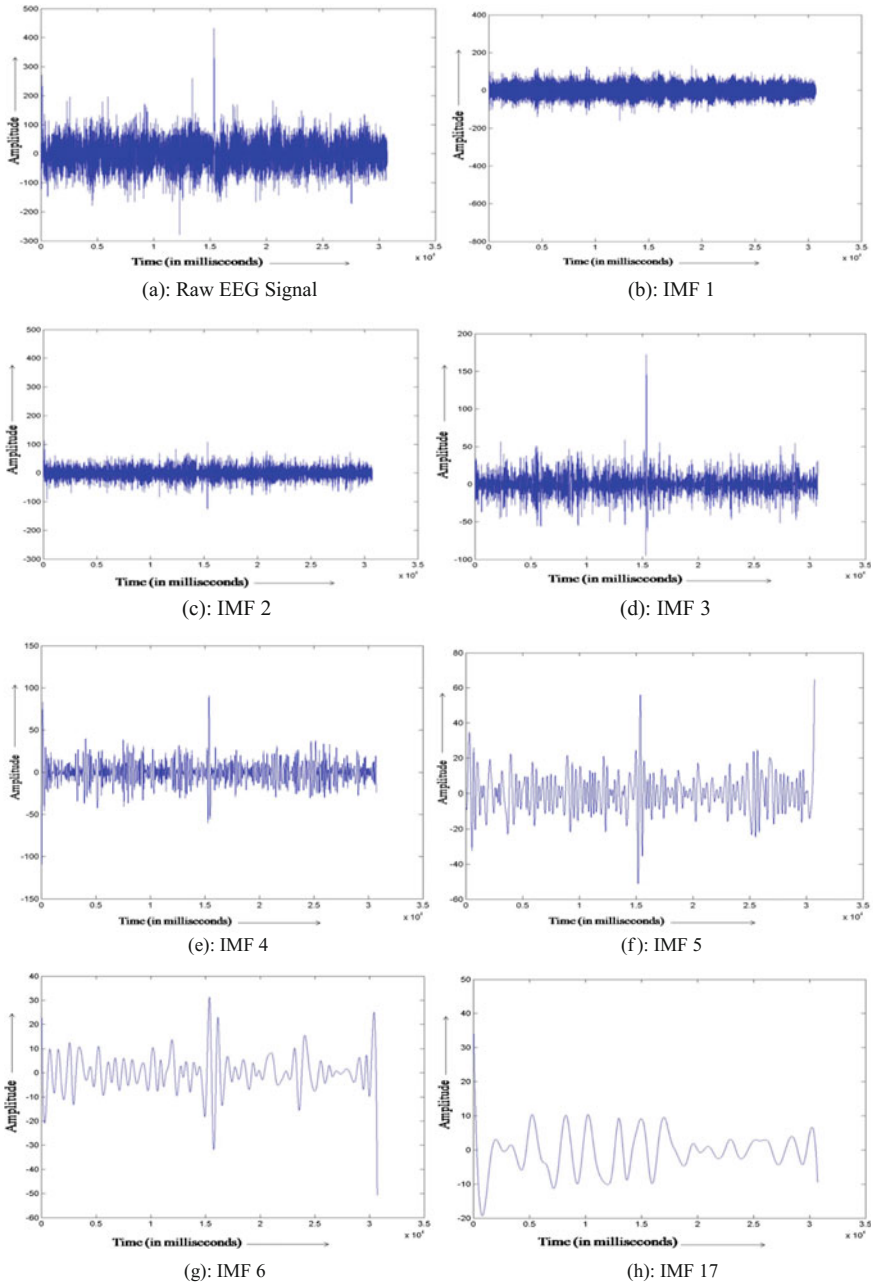


Fig. 2.1 Empirical Mode decomposition of a 10 s 'with drone' EEG signal of F3 electrode

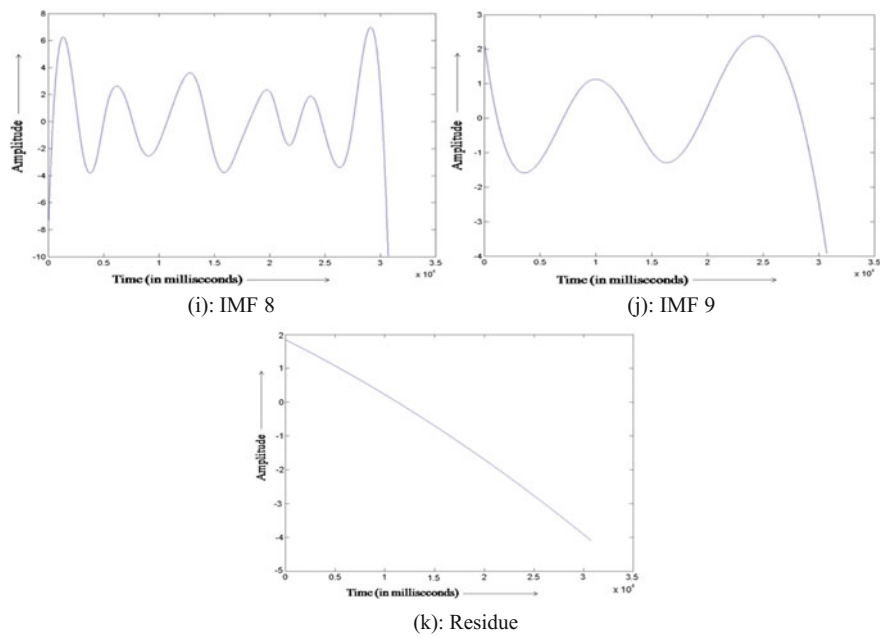


Fig. 2.1 (continued)

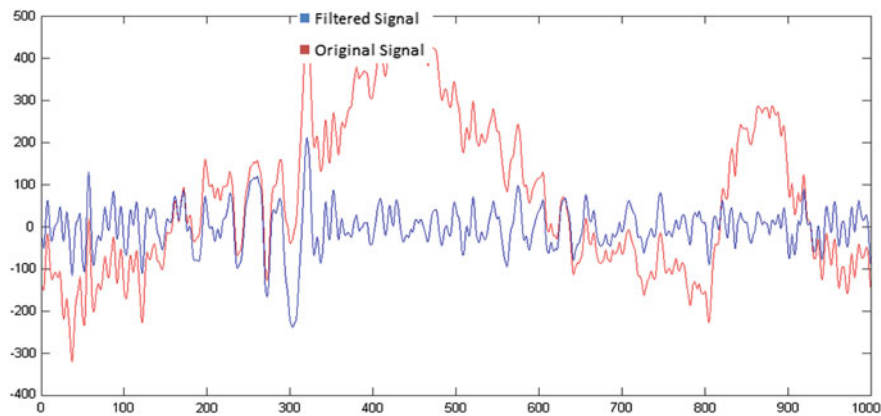


Fig. 2.2 Raw EEG signal and Artifact free EEG signal of 10 s

filter $g[n]$ and a low pass filter $h[n]$. This constitutes one level of decomposition and can mathematically be expressed as follows:

$$y_{high}[k] = \sum x[n].g[2k - n] \quad (2.11)$$

$$y_{low}[k] = \sum x[n].h[2k - n] \quad (2.12)$$

where $y_{high}[k]$ and $y_{low}[k]$ are the outputs of the high pass and low pass filters respectively, after sub sampling by 2. This decomposition halves the time resolution since only half the number of samples now characterizes the entire signal. However, this operation doubles the frequency resolution, since the frequency band of the signal now spans only half the previous frequency band, effectively reducing the uncertainty in the frequency by half. The above procedure, which is also known as the sub band coding, can be repeated for further decomposition (Sivanandam and Deepa 2006; Mehrotra et al. 1997). Using the DWT technique we have extracted the amplitude envelope as well as the time series data corresponding to the two different experimental conditions for all the frontal electrodes.

The amplitude envelope of alpha and theta frequency ranges have been obtained for all the frontal electrodes in “before drone” and “with drone” conditions. The time series data for the alpha and theta frequency ranges have also been obtained. Figures 2.3 and 2.4 are representative figures which demonstrate the change of alpha and theta frequency rhythms under the application of drone music (Maity et al. 2015). The figure shows a 10 s EEG alpha and theta rhythm for F3 electrode in the two experimental conditions. A definite response is reflected in both the low frequency ranges under the application of drone.

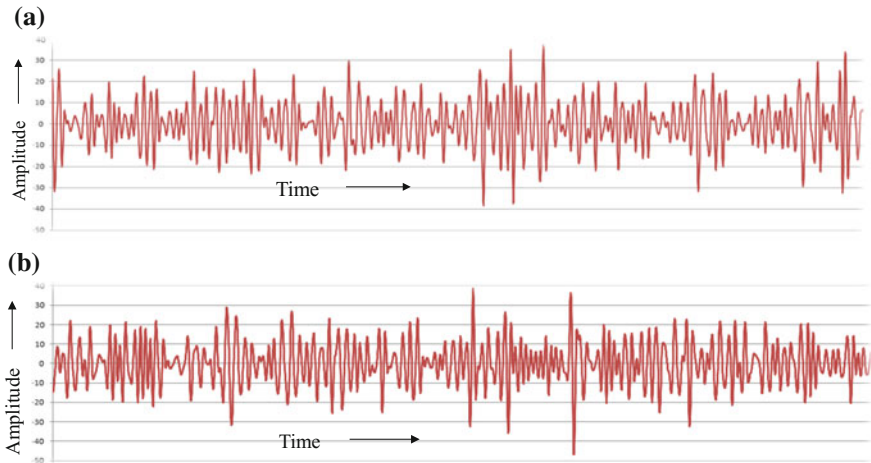


Fig. 2.3 10 s alpha frequency range data for **a** “before drone” and **b** “with drone” condition

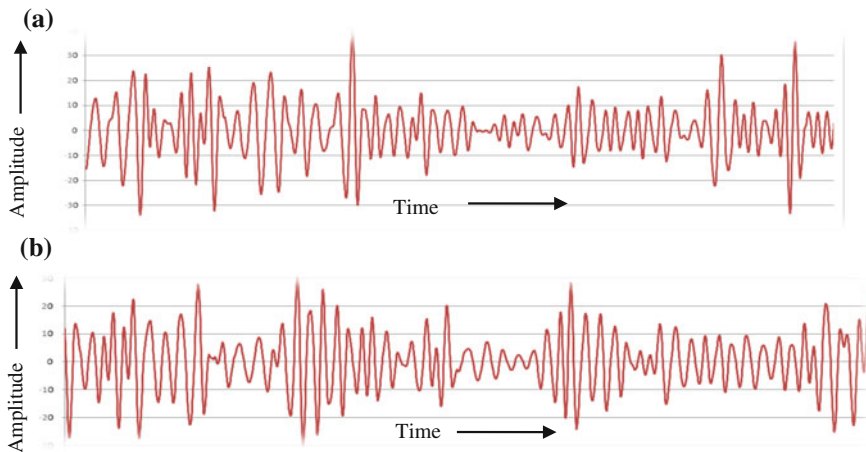


Fig. 2.4 10 s theta frequency range data for **a** “before drone” and **b** “with drone” condition

The obtained data from WT technique is subjected to DFA/MFDFA/MFDXA methods while sometimes the complete EEG signal has also been subjected to these techniques.

2.4 Detrended Fluctuation Analysis

Detrended Fluctuation Analysis (DFA) is used to analyze the long range temporal correlations (LRTC) of the observed fluctuations in EEG. In the realm of complex cognition, scaling analysis technique was used to confirm the presence of universality and scale invariance in spontaneous EEG signals (Linkenkaer-Hansen et al. 2001; Peng et al. 1994). In stochastic processes, chaos theory and time series analysis, DFA is a method for determining the statistical self-affinity of a signal. It is useful for analyzing time series that appear to be long-memory processes (diverging correlation time, e.g. power-law decaying autocorrelation function) or $1/f$ noise. The obtained exponent is similar to the Hurst exponent, except that DFA may also be applied to signals whose underlying statistics (such as mean and variance) or dynamics are non-stationary (changing with time). DFA method was applied in (Karkare et al. 2009) to show that scale-free long-range correlation properties of the brain electrical activity are modulated by a task of complex visual perception, and further, such modulations also occur during the mental imagery of the same task. In case of music induced emotions, DFA was applied to analyze the scaling pattern of EEG signals in emotional music (Gao et al. 2007) and particularly Indian music (Banerjee et al. 2016). The DFA of a time series $[x_1, x_2, \dots, x_N]$ are as follows.

Step 1: Converting the noise like structure of the signal into a random walk like signal. It can be represented as:

$$Y(i) = \sum (x_k - \bar{x}) \quad (2.13)$$

where $\bar{x}$ is the mean value of the signal.

Step 2: The whole length of the signal is divided into N_s number of segments consisting of certain no. of samples. For s as sample size and N the total length of the signal the segments are

$$N_s = \text{int}\left(\frac{N}{s}\right) \quad (2.14)$$

The original signal with the extracted trends has been shown in the Figs. 2.5 and 2.6 given at the end of this section.

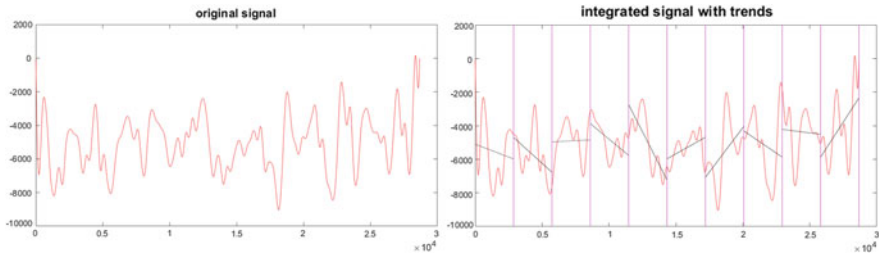


Fig. 2.5 Raw signal with the polyfit trends as found in Steps 1 and 2

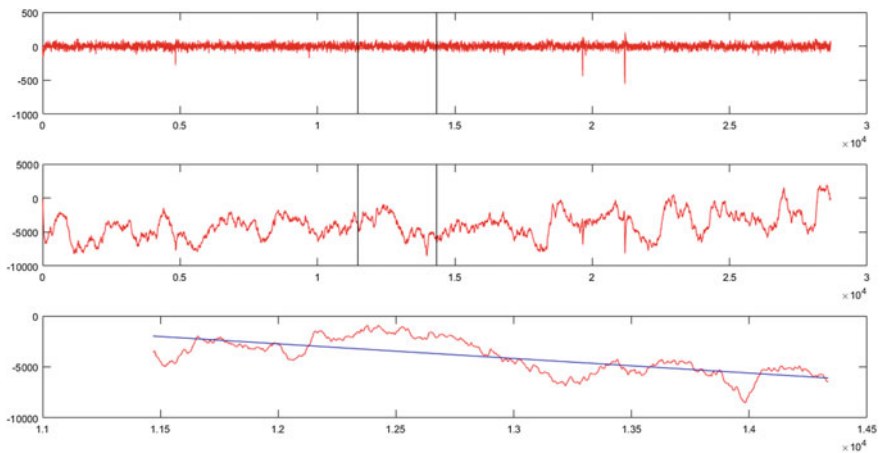


Fig. 2.6 Complete Signal with poly-fit trends

Step 3: The local RMS variation for any sample size s is the function $F(s, v)$. This function can be written as follows:

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{Y[(v-1)s + i] - y_v(i)\}^2 \quad (2.15)$$

Step 4: The q -order overall RMS variation for various scale sizes can be obtained by the use of following equation

$$F_q(s) = \left\{ \frac{1}{Ns} \sum_{v=1}^{Ns} [F^2(s, v)]^{\frac{q}{2}} \right\}^{\left(\frac{1}{q}\right)} \quad (2.16)$$

Step 5: The scaling behaviour of the fluctuation function is obtained by drawing the log-log plot of $F_q(s)$ versus s for each value of q .

$$F_q(s) \sim s^{h(q)} \quad (2.17)$$

where $h(q)$ is called the generalized Hurst exponent expressed as the slope of a double logarithmic plot (Fig. 2.7). For $q = 2$, we obtain the monofractal scaling exponent or α . A monofractal time series is characterized by unique $h(q)$ for all values of q . The parameter α (scaling exponent, autocorrelation exponent, self-similarity parameter etc.) represents the autocorrelation properties of the signal. DFA technique was applied following the NBT algorithm used in Hardstone et al. (2012). The scaling exponent provides a quantitative measure of long range temporal correlation (LRTC) that exists in the EEG. When the EEG is completely uncorrelated (Gaussian or non-Gaussian probability distribution), the calculation of the scaling exponent yields 0.5, also called “white noise”.

When applied to EEG data with LRTC, power-law behavior will generate scaling exponents with greater than 0.5 and less than 1. As the scaling exponent increases from 0.5 to 1, the LRTC in the EEG are more persistent (decaying more slowly with time). If a scaling exponent is greater than 1, the LRTC no longer exhibits power law behavior. Finally, if the scaling exponent = 1.5, this indicates Brownian noise, which is the integration of white noise. It can be converted into the Hurst exponent $H = \alpha - 1$ and the estimated FD accordingly as

$$D_{\text{DFA}} = 3 - \alpha. \quad (2.18)$$

The FD values were computed from Eq. (2.18) for all the frequency rhythms.

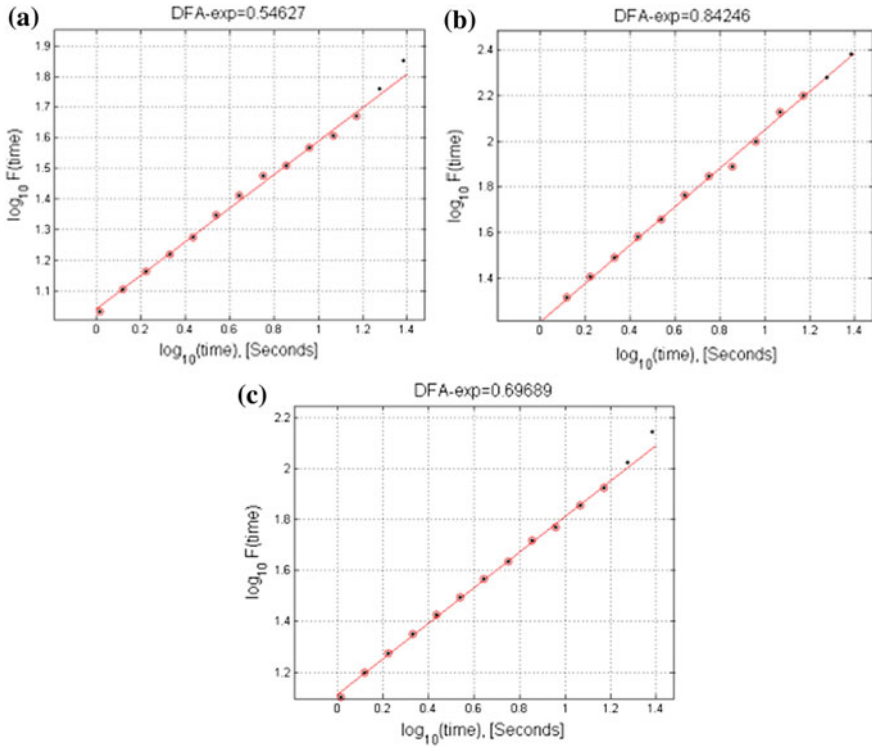


Fig. 2.7 a–c A sample log-log plot of alpha wave of an EEG signal in the three different experimental conditions

2.5 Multifractal Detrended Fluctuation Analysis (DFA)

The real-life fractal patterns that we see hardly scale according to a single scaling exponent, rather there should be multiple scaling laws to capture their growth or variation over time. These spatial and temporal scale variations indicate a multifractal structure of a particular signal that is defined by a multifractal spectrum of power law exponents. For these more practical cases, Kantelhardt et al. (2002) formulated the MFDFA algorithm which is essentially a generalization of the DFA algorithm as given before but takes into account different scaling ratios. For Eq. (2.17), putting $q = 2$, the standard DFA procedure is retrieved. We are interested in how the generalized q dependent fluctuation functions $F_q(s)$ depend on the time scale s for different values of q . Hence, we must repeat steps 2–4 for several time scales s . It is apparent that $F_q(s)$ will increase with increasing s . Of course, $F_q(s)$ depends on the DFA order m . By construction, $F_q(s)$ is only defined for $s \geq m + 2$. Again Step 5 is repeated with different values of q ;

Step 5: Determination of the scaling behavior of the fluctuation functions by analyzing log-log plots $F_q(s)$ versus s for each value of q . If the series x_i are

long-range power-law correlated, $F_q(s)$ increases, for large values of s , as a power-law,

$$F_q(s) \sim s^{h(q)} \quad (2.17)$$

In general, the exponent $h(q)$ may depend on q . For stationary time series, $h(2)$ is identical to the well-known Hurst exponent H . Thus, we will call the function $h(q)$ generalized Hurst exponent.

The generalized Hurst exponent $h(q)$ of MFDFA is related to the classical scaling exponent $\tau(q)$ by the relation

$$\tau(q) = qh(q) - 1 \quad (2.19)$$

A monofractal series with long range correlation is characterized by linearly dependent q order exponent $\tau(q)$ with a single Hurst exponent H . Multifractal signal on the other hand, possess multiple Hurst exponent and in this case, $\tau(q)$ depends non-linearly on q (Ashkenazy et al. 2003).

The singularity spectrum $f(\alpha)$ is related to $h(q)$ by

$$\alpha = h(q) + qh'(q) \quad (2.20)$$

$$f(\alpha) = q[\alpha - h(q)] + 1 \quad (2.21)$$

where α denoting the singularity strength and $f(\alpha)$, the dimension of subset series that is characterized by α . The width of the multifractal spectrum essentially denotes the range of exponents. The spectra can be characterized quantitatively by fitting a quadratic function with the help of least square method (Figliola et al. 2007) in the neighbourhood of maximum,

$$f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C \quad (2.22)$$

Here C is an additive constant $C = f(\alpha_0) = 1$ and B is a measure of asymmetry of the spectrum. So obviously it is zero for a perfectly symmetric spectrum. We can obtain the width of the spectrum very easily by extrapolating the fitted quadratic curve to zero.

Width W is defined as,

$$W = \alpha_1 - \alpha_2$$

with

$$f(\alpha_1) = f(\alpha_2) = 0$$

The width of the spectrum gives a measure of the multifractality of the spectrum. Greater is the value of the width W greater will be the multifractality of the spectrum. For a monofractal time series, the width will be zero as $h(q)$ is independent of q .

The origin of multifractality in a EEG time series can be verified by randomly shuffling the original time series data (Figliola et al. 2007). In general, two different types of multifractality are present in a time series data: (i) Multifractality due to a broad probability density function for the values of the time series. Here, the multifractality of the time series cannot be removed by random shuffling and the shuffled data has the same variation of $h(q)$ as the original data (ii) Multifractality due to a variety of long-range correlations due to the small and large fluctuations. In this case, the probability density function of the values can be a regular distribution with finite moments, for e.g. a Gaussian distribution. The corresponding shuffled series will exhibit non-multifractal scaling, since all long-range correlations are destroyed by the shuffling procedure. All long range correlations that existed in the original data are removed by this random shuffling and what remains is a totally uncorrelated sequence. Hence, if the multifractality of the original data was due to long range correlation, the shuffled data will show non-fractal scaling. If any series has multifractality both due to long range correlation as well as due to probability density function, then the shuffled series will have smaller width W and hence weaker multifractality than the original time series.

The q th order fluctuation function $Fq(s)$ for 10 points of q in between -5 and $+5$ was obtained. The time series values of both the waves have been randomly shuffled to destroy all the long range correlations present in the data, and what remained is a totally uncorrelated sequence. The regression plot of $\ln(Fq(s))$ versus $\ln(s)$ averaged for different values of q ($q = -3$ to $q = +3$) is shown in the plot for scales varying from 16 to 1024) for a sample electrode F3 is given in Fig. 2.8(a–d) for both alpha and theta waves. The slope of the best fit line thus obtained from $\ln(Fq(s))$ versus $\ln(s)$ plot gives the values of $h(q)$. It is seen from Fig. 2.8 that the

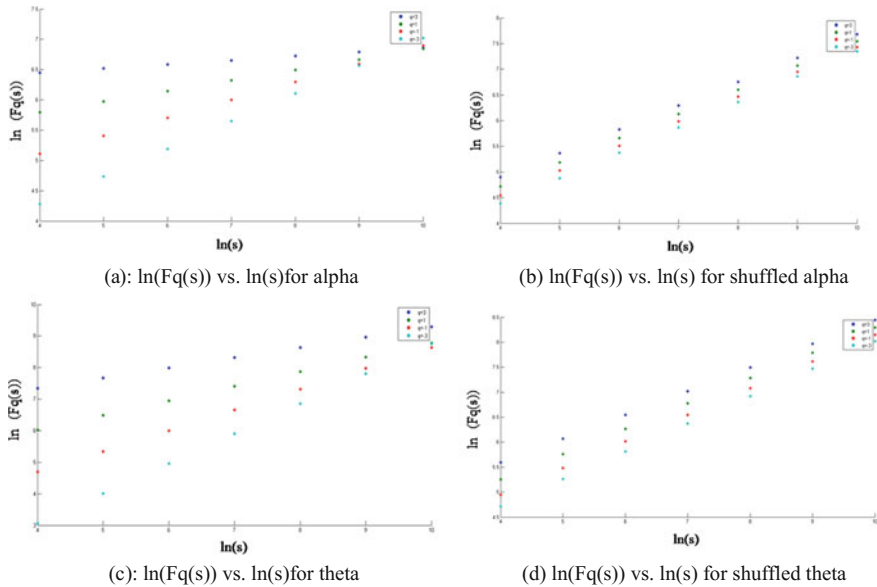


Fig. 2.8 a–d Plot of $\ln(Fq(s))$ versus $\ln(s)$ showing different $h(q)$ corresponding to each q

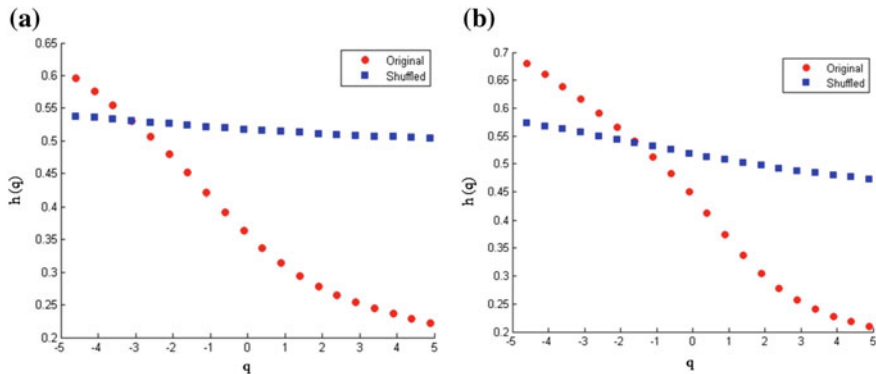


Fig. 2.9 Variation of $h(q)$ with q for original and shuffled series

shuffled values of both α and θ do not change with the values of q , and thus has a fixed slope $h(q) = H$, which is the conventional Hurst exponent for monofractal time series.

For monofractal time series, $h(q)$ is independent of q , since the scaling behavior of the variances $F^2(s, v)$ is identical for all segments v , and the averaging procedure in Eq. (2.18) will give just this identical scaling behavior for all values of q , only if small and large fluctuations scale differently, there will be a significant dependence of $h(q)$ on q . For positive values of q , $h(q)$ describes the scaling behavior of the segments with large fluctuations. Usually the large fluctuations are characterized by a smaller scaling exponent $h(q)$ for multifractal series. On the contrary, for negative values of q , the segments v with small variance $F^2(s, v)$ will dominate the average $Fq(s)$. Hence, for negative values of q , $h(q)$ describes the scaling behavior of the segments with small fluctuations, which are usually characterized by a larger scaling exponent.

A representative figure for variation of $h(q)$ with q for two different time-series is shown in Fig. 2.9a, b. It is clearly evident from the figures that the values of $h(q)$ decreases with the increase of q , showing multifractal scaling in both the signals. For monofractal signals, a single value of Hurst exponent is obtained corresponding to different values of q , like the shuffled value of $h(q)$ as seen in both the figures, where $h(q)$ remains almost constant with the change of q . The amount of multifractality can be determined quantitatively in each of the windows of each signal from the width of the multifractal spectrum [$f(\alpha)$ vs. α]. The shuffled width obtained, is found to be always smaller than the original width of the signal (Fig. 2.10). This ascertains the fact that multifractality in the signals is both due to long range correlations as well as broad probability density function. In the ideal case, the shuffled data should behave as a monofractal signal with no multifractal scaling. Thus, in the plot of Hurst exponent, it is seen that the shuffled values of $h(q)$ does not change in general with q , and in the $f(\alpha)$ versus α plot, the shuffled series will show a peak at α_0 close to 0.5. A representative figure (Fig. 2.11) shows the $f(\alpha)$ versus α plot for a single person in the α and θ frequency range for the two experimental conditions.

As a generalization of the DFA method, the detrended cross-correlation analysis (DCCA) is proposed to investigate the long-term cross-correlations between two

Fig. 2.10 $f(\alpha)$ versus α curve for original and shuffled series

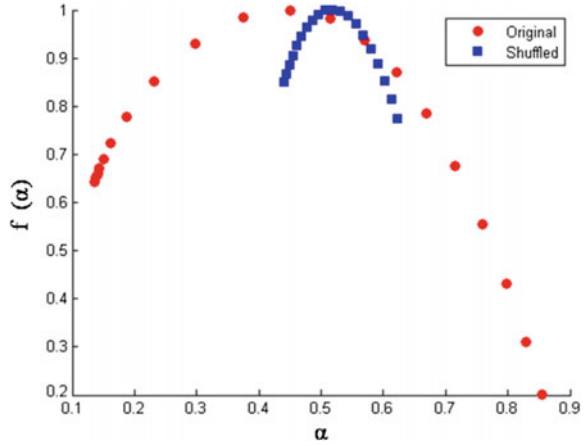
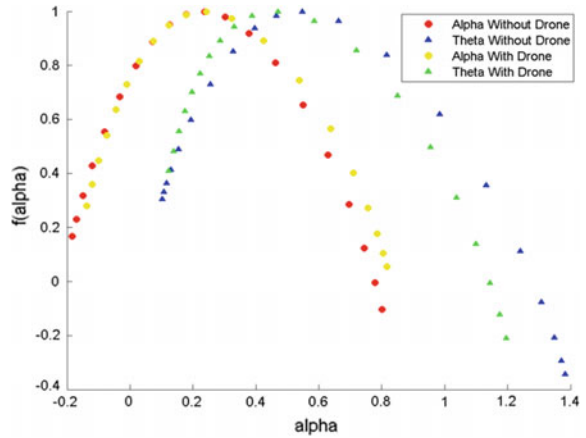


Fig. 2.11 Variation of spectral width in alpha and theta domain



non stationary time series (Podobnik et al. 2008, 2009; Podobnik and Stanley 2008; Xu et al. 2010), and Multifractal Detrended Cross-Correlation Analysis (MF-DXA) can unveil the multifractal features of two cross-correlated signals (He and Chen 2011; Jiang and Zhou 2011; Wang et al. 2013; Ghosh et al. 2014).

2.6 Multifractal Detrended Cross-Correlation Analysis (MFDXA)

MFDXA method was first used by Zhou (2008) and is an offshoot of the generalized MFDFA method. Here, we compute the profiles of the underlying data series $x(i)$ and $y(i)$ as

$$\begin{aligned} X(i) &\equiv \left[\sum_{k=1}^i x(k) - x_{\text{avg}} \right] & \text{for } i = 1 \dots N \\ Y(i) &\equiv \left[\sum_{k=1}^i x(k) - x_{\text{avg}} \right] & \text{for } i = 1 \dots N \end{aligned} \quad (2.23)$$

The next steps proceed in the same way as the MFDFA method, with the only difference being we have to take $2N_s$ bins here. The q th order detrended covariance $F_q(s)$ is obtained after averaging over $2N_s$ bins.

$$F_q(s) = \left\{ 1/2 N_s \sum_{v=1}^{2N_s} [F(s, v)]^{q/2} \right\}^{q/2} \quad (2.24)$$

where q is an index which can take all possible values except zero because in that case the factor $1/q$ blows up. The procedure can be repeated by varying the value of s . $F_q(s)$ increases with increase in value of s . If the series is long range power correlated, then $F_q(s)$ will show power law behavior

$$F_q(s) \sim s^{\lambda(q)}.$$

If such a scaling exists $\ln F_q$ will depend linearly on $\ln s$, with $\lambda(q)$ as the slope. Scaling exponent $\lambda(q)$ represents the degree of the cross-correlation between the two time series. In general the exponent $\lambda(q)$ depends on q . We cannot obtain the value of $\lambda(0)$ directly because F_q blows up at $q = 0$. F_q cannot be obtained by the normal averaging procedure; instead a logarithmic averaging procedure is applied

$$F_0(s) = \left\{ 1/2 N_s \sum_{v=1}^{2N_s} [F(s, v)] \right\} \sim s^{\lambda(0)} \quad (2.25)$$

For $q = 2$ the method reduces to standard DCCA. If scaling exponent $\lambda(q)$ is independent of q , the cross-correlations between two time series are monofractal. If scaling exponent $\lambda(q)$ is dependent on q , the cross-correlations between two time series are multifractal. Furthermore, for positive q , $\lambda(q)$ describes the scaling behavior of the segments with large fluctuations and for negative q , $\lambda(q)$ describes the scaling behavior of the segments with small fluctuations. Scaling exponent $\lambda(q)$ represents the degree of the cross-correlation between the two time series $x(i)$ and $y(i)$. The value $\lambda(q) = 0.5$ denotes the absence of cross-correlation. $\lambda(q) > 0.5$ indicates persistent long range cross-correlations where a large value in one variable is likely to be followed by a large value in another variable, while the value $\lambda(q) < 0.5$ indicates anti-persistent cross-correlations where a large value in one

variable is likely to be followed by a small value in another variable, and vice versa (Movahed and Hermanis 2008).

Zhou (2008) found that for two time series constructed by binomial measure from p-model, there exists the following relationship:

$$\lambda(q = 2) \approx [h_x(q = 2) + h_y(q = 2)]/2 \quad (2.26)$$

Podobnik and Stanley have studied this relation when $q = 2$ for monofractal Autoregressive Fractional Moving Average (ARFIMA) signals and EEG time series (Podonik and Stanley 2008).

In case of two time series generated by using two uncoupled ARFIMA processes, each of both is autocorrelated, but there is no power-law cross correlation with a specific exponent (Movahed and Hermanis 2008). According to auto-correlation function given by:

$$C(\tau) = \langle [x(i + \tau) - \langle x \rangle][x(i) - \langle x \rangle] \rangle \sim \tau^{-\gamma} \quad (2.27)$$

The cross-correlation function can be written as

$$C_x(\tau) = \langle [x(i + \tau) - \langle x \rangle][y(i) - \langle y \rangle] \rangle \sim \tau_x^{-\gamma} \quad (2.28)$$

where γ and γ_x are the auto-correlation and cross-correlation exponents, respectively. Due to the non-stationarities and trends superimposed on the collected data, direct calculation of these exponents are usually not recommended; rather the reliable method to calculate auto-correlation exponent is the DFA method, namely $\gamma = 2 - 2h$ ($q = 2$) (Movahed and Hermanis 2008). Recently, Podobnik et al. (2011), have demonstrated the relation between cross-correlation exponent, γ_x and scaling exponent $\lambda(q)$ derived by Eq. (2.2) according to $\gamma_x = 2 - 2\lambda$ ($q = 2$). For uncorrelated data, γ_x has a value 1 and the lower the value of γ and γ_x more correlated is the data. In general, $\lambda(q)$ depends on q , indicating the presence of multifractality. In other words, we want to point out how two non-linear signals are cross-correlated in various time scales.

The q th order detrended covariance $F_q(s)$ was obtained from relations 3 and 4 for values of q from -5 to $+5$ in steps of 1 just like the MFDFA part. Power law scaling of $F_q(s)$ with s is observed for all values of q as is seen from Fig. 2.8a–d same as those found for MFDFA. We have also shown variation of $h(q)$ with q for Part 1 of four clips by means of MF-DFA in Fig. 2.9. The plot depicts multifractal behavior of cross-correlations because for different q , there are different exponents; that is, for different q , there are different power-law cross-correlations. Further from the same figure we can see that the value of $H(q)$ depends on q for all the four samples that we have taken in this study. We know that $H(q) = 0.5$ indicates that the series is an independent random process, and for $H(q) < 0.5$ it is characterized by long-range anti-correlations while for $0.5 < H(q) < 1$, it is featured by long-term

correlations. In this case the signal is stationary. The exponent H ($q = 2$) is equivalent with the well-known Hurst index. A representative figure (Fig. 2.12) reports the variation of cross correlation exponent $\lambda(q)$ with q for two particular samples (Part 1 for Sample 1 and Sample 2), also the variation of $h(q)$ with q for those two samples obtained from MFDFA technique are also shown in the same figure for comparison.

The variation of $\lambda(q)$ with q for the two cross correlated signals (Part 1 for Sample 1 and Sample 2) show that they are multifractal in nature (Fig. 2.12). To illustrate further the presence of multifractality in the cross-correlated music signals, i.e. to have information about the distribution of degree of cross-correlation in various time scales, a representative multifractal spectrum was plotted for the two signals in Fig. 2.13. The way to characterize multifractality of cross correlation between two samples is to relate via a $\lambda(q)$ Legendre Transform as in the case of

Fig. 2.12 Variation of $\lambda(q)$ and $h(q)$ for two sound signals

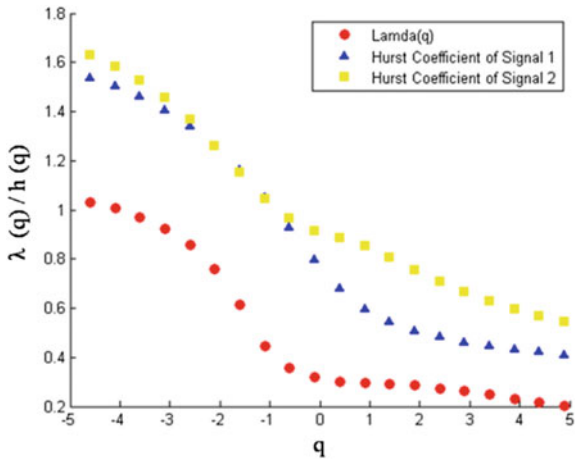
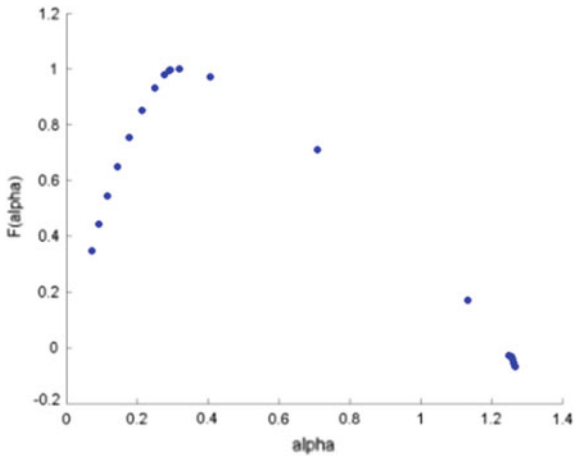


Fig. 2.13 Multifractal cross-correlated Spectrum of samples 1 and 2 (Part 1)



single series (Feder 2013). The growth of the width of $f(\alpha)$ or equivalently $\Delta\alpha$ shows the increase in degree of multifractality of the coupled signals. Again, it becomes evident from the spectrum that the cross correlated signals are multifractal in various time scales.

Jones and Kaul (1996) were the first to reveal a stable negative cross-correlation between oil prices and stock prices. The negative cross-correlations were also found in a number of previous works (Chen 2009; Berument et al. 2010; Reboredo et al. 2014). A negative value of cross correlation is an indication of strong cross-correlation between the two samples for which the cross correlation is being carried out. Using the MFDXA technique we have estimated the degree of cross-correlations between neuronal potentials originating from different lobes of human brain as well as parts of musical clip.

In the next section, we proceed to propose novel algorithms for feature extraction from raw EEG data which may open up new vistas in developing an automated emotion classification algorithm from music induced EEG signal analysis.

2.7 Estimation of Neural Jitter and Shimmer

Jitter conventionally refers to the variability of fundamental frequency while shimmer refers to the variability in the peak to peak amplitude (Farrús and Hernando 2009). These parameters have long been used by scientists working on speech and music signal processing for characterization of a speech/music signal. In speech signal processing, jitter/shimmer is a measure for vocal stability of a person. A jitter of less than 1% and a shimmer of less than 7% in frequency/amplitude is considered for a normal person, which increases/decreases in case of voice disorder. EEG signals possess almost the same properties in temporal domain like music/speech signal. Hence, applying the same concept to neural EEG domain, we have termed this factor as neural-jitter and neural-shimmer for EEG signals. These parameters are being calculated essentially to measure the perturbation index of a signal. We started off with calculation of neural jitter and shimmer percentage for different types of stimuli induced EEG signals and compared them with rest-state EEG signals. In this way we look to develop a threshold value/percentage like the one in speech signal which can objectively help in the assessment of mental state of a person. EEG signals (Fig. 2.14) are obtained in the temporal domain showing the variation of Extracellular Field Potentials (Buzsáki et al. 2012) with respect to time. Digital Signal processing methods allows converting between temporal domains to frequency domain. The transform is called Fourier Transform (Bracewell 1965) which generates Fourier coefficient for each frequency present in the temporal signal. The plot of the frequency domain, which shows the Fourier coefficients, is called frequency spectrum, (Fig. 2.15). The work is aimed to look into proposing a novel technique to calculate traditional Jitter/Shimmer in neural signals. The EEG signals were transformed into frequency domain double sided frequency spectrum

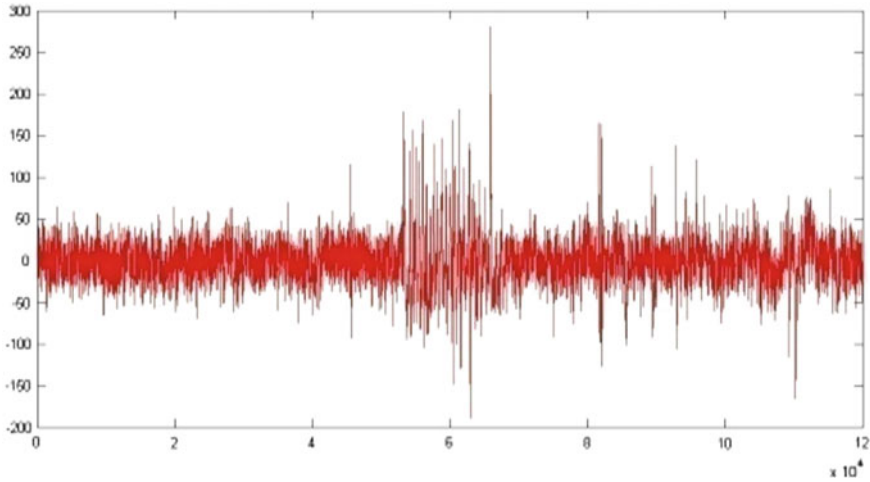


Fig. 2.14 Sample EEG signal plotted against y and x axes where y represents the values of the signal at different times 't' and x represents the time 't' axis

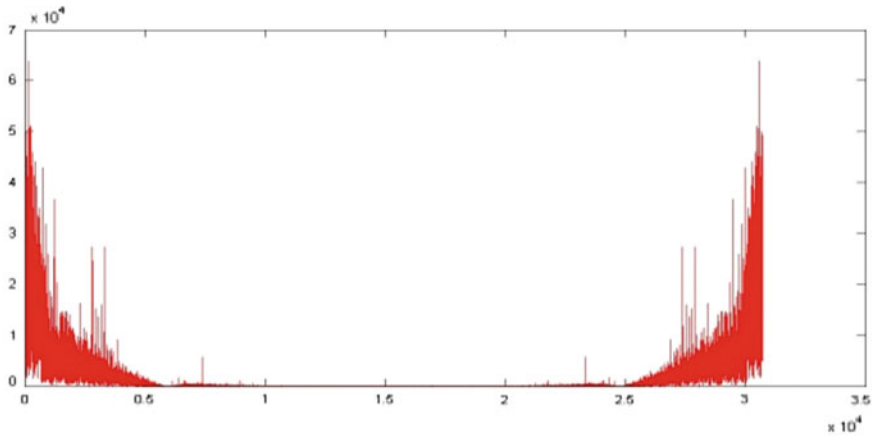


Fig. 2.15 Two-sided amplitude spectrum

(Fig. 2.16). The Fourier coefficients can be both real and complex. The plot and analysis was made with the absolute values of the coefficients. Single sided spectrum (Fig. 2.17) was used for analysis instead of double sided. An attempt to analyze the single sided frequency spectrum is made by considering only top 20% frequency coefficients. Threshold is 20% of ($A_{\max}$) where is $A_{\max}$ the largest Fourier coefficients (Fig. 2.18). The Fourier coefficients above $A_{\max}/5$ continue to posses

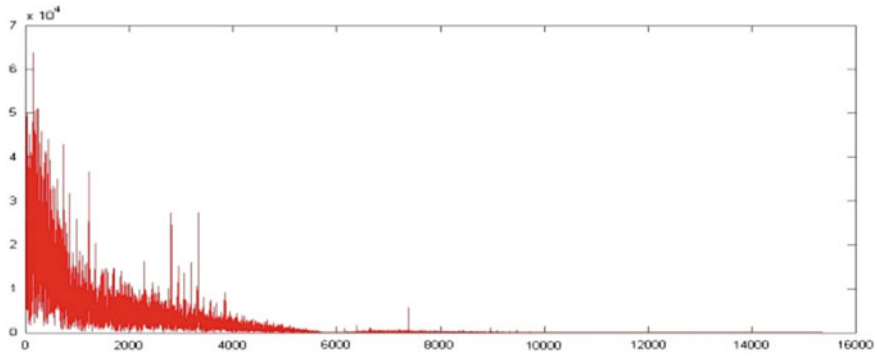


Fig. 2.16 Single-sided amplitude spectrums

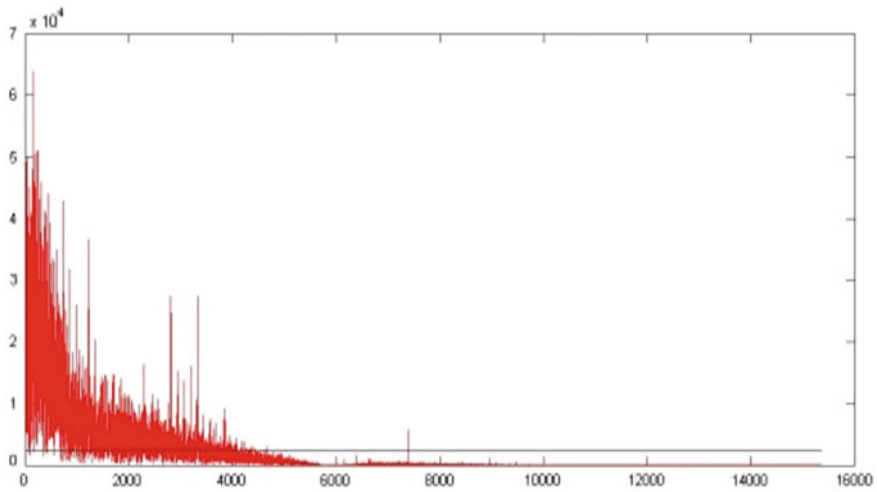


Fig. 2.17 Thresholding operations

its values and below $A_{\max}/5$ becomes zero (Fig. 2.19). This transformation is expected to be good as values less than $A_{\max}/5$ are literally less so losing those coefficient is expected not to cause much loss of information. A Frequency thresholding (FT) matrix is computed storing values of time periods ($2 * \pi * \omega$), ω are the frequencies with non zero Fourier coefficients in thresholding curve and the corresponding fourier coefficients. Figure 2.19 shows the plot with non-zero coefficients only. The time elements in FT matrix serve as T_i in calculating the jitter and the corresponding coefficient elements in FT matrix serve as A_i in the shimmer calculation.

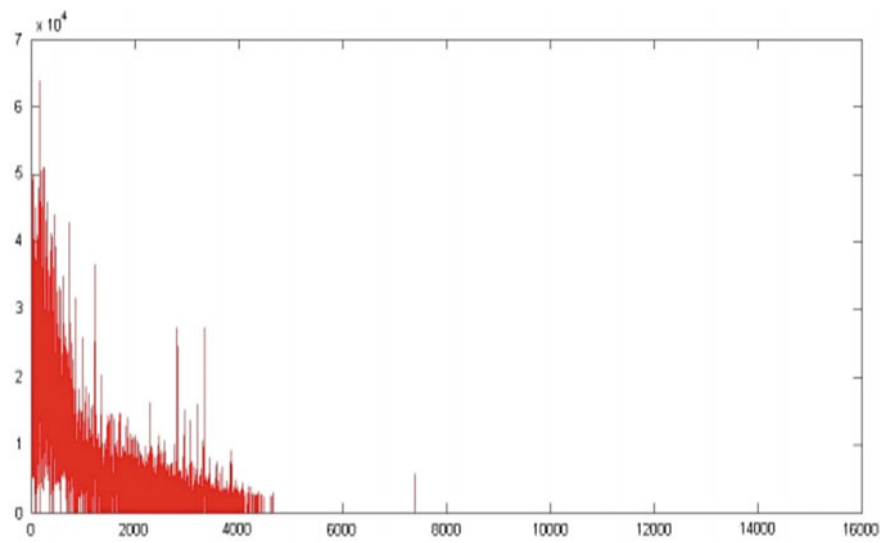


Fig. 2.18 Single-sided amplitude spectrums of the signal after thresholding with zero Fourier coefficients

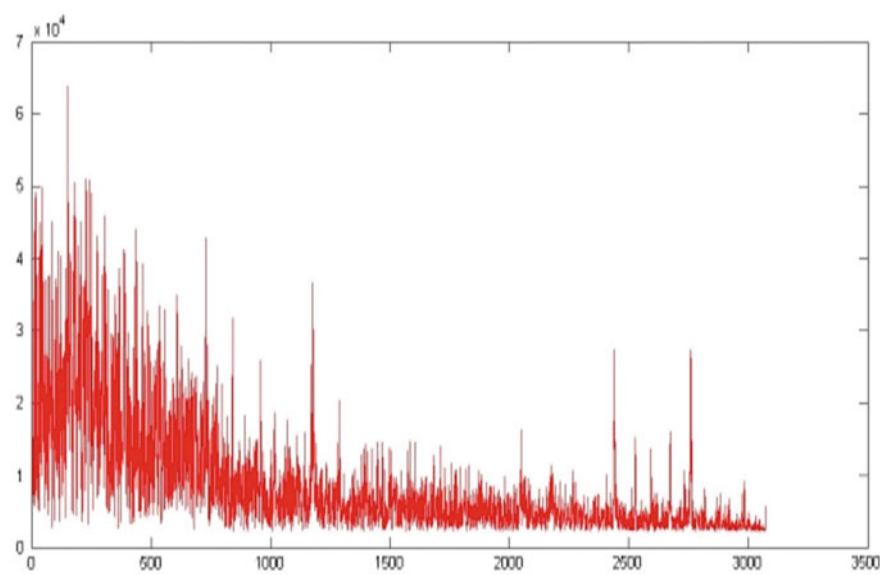


Fig. 2.19 Single-sided amplitude spectrums of the signal after thresholding with non-zero Fourier coefficients

Measurement of Neural Shimmer Values:

Shimmer is the variation of peak-to-peak amplitude. It is defined as the mean absolute difference between amplitudes of successive periods divided by the mean value of the amplitudes. Mathematically, given as

$$\frac{\frac{1}{N-1} \sum_{i=1}^{N-1} |A_i - A_{i+1}|}{\frac{1}{N} \sum_{i=1}^N A_i}$$

where N is the number of periods and A_i is the amplitude.

Measurement of Neural Jitter Values:

Jitter is the perturbation of fundamental frequency. It can also be defined as the deviation from true periodicity of an apparently periodic signal, given as the mean absolute difference between successive periods divided by the mean value of the periods. Mathematically, given as

$$\frac{\frac{1}{N-1} \sum_{i=1}^{N-1} |T_i - T_{i+1}|}{\frac{1}{N} \sum_{i=1}^N T_i}$$

where N is the number of periods and T_i is the duration (in seconds) of the i th period.

The jitter and shimmer values essentially gives an estimate of the perturbations in the amplitude/frequency of the EEG signals. These perturbations when computed as a function of time will give a time series which is essentially record of the fluctuation of the fluctuations. Non-linear scaling methods detailed above can be used for scaling the fluctuation property of jitter/shimmer time series.

2.8 Estimation of Pitch of EEG Signal from Zero-Crossings

EEG signal comprises of a number of spectral components. The lower and upper cut-offs of the frequency range of an EEG signal is indistinguishable, essentially ranging from 0.5 to 30 Hz. As has been iterated before, EEG rhythms are categorized as follows: Delta (0–4 Hz), Theta (4–7 Hz), Alpha (8–13 Hz), Beta (14–30 Hz) and Gamma (30–50 Hz). We use the DWT algorithm explained in this chapter for decomposing an EEG signal into these frequency bands first. The EEG sampling frequency being 256 Hz in our case. We considered 3 signals containing respectively the 3 frequency bands, namely, Alpha, Theta and Gamma obtained from wavelet decomposition of a particular EEG signal. Suppose ‘X’ denotes the signal containing frequencies in the Alpha band. The recorded time of each signal was 2 min, i.e., 120 s. So, we attempt to divide ‘X’ into intervals of length 1 s, the

total number of segments being 120. In each of these segments we calculated the number of zero-crossings, that is, the number of times the curve cuts the time axis. We denote the number of zero-crossings for each interval ' i ' as Z_i , since the value changes with the interval number. The number of zero crossing or the zero crossing rate (ZCR) is an indirect way of measuring the fundamental frequency which has been used in a number of studies (Keddeem 1986; Scheirer and Slaney 1997; Roads 1996) in the acoustic domain. Using the same technique, we sought to have an idea of the neural pitch of an EEG signal. For a particular signal (say F4 electrode, Alpha band signal for Subject 1 in resting state) we have a *Pitch* matrix denoted as ' P ' which essentially contains each of Z_i for each interval i where i ranges from 1 to 120. The size of this P matrix is 1×120 (a row matrix containing the number of columns = number of segments as mentioned previously). For a particular electrode for 3 frequency band signals we concatenate the matrix P which now becomes 3×120 . Using MATLAB we show scatter plots of Z_i as a function of i . Figure 2.20a–c denotes the variation of ZCR in F3 electrode under the effect of a pair of music of contrast emotion. While Fig. 2.20a shows the distribution in rest condition, Fig. 2.20b, c denote the distribution of zero crossing under the effect of happy and sad music clip respectively.

A simple glance to the figures reveal that the distribution pattern significantly changes under the effect of two different categories of music clips. The probability distribution plot of different fundamental frequencies is expected to reveal unique information about the processing of different emotions in human brain. The same has been repeated for theta and gamma frequency ranges and analyzed rigorously to

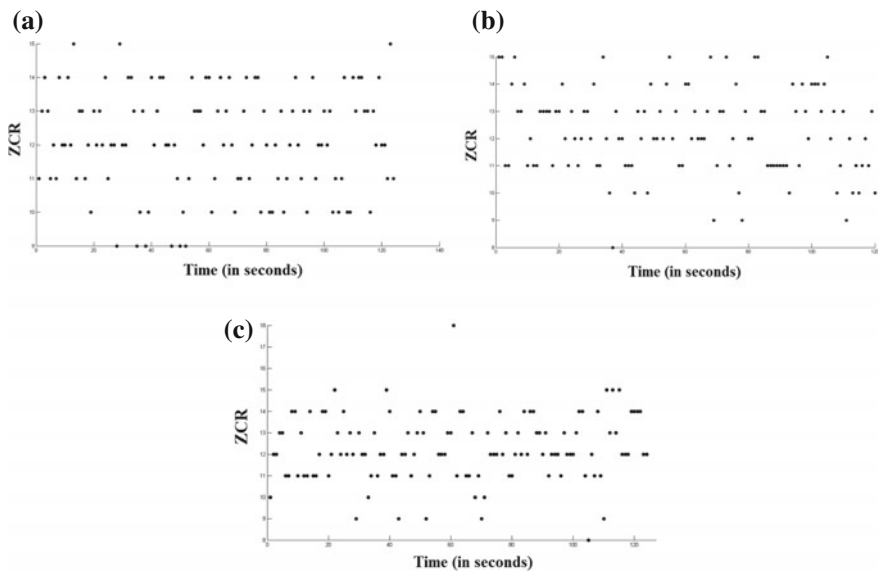


Fig. 2.20 a ZCR in rest condition, b ZCR in happy music, c ZCR in sad music

find the existence of a preferred fundamental under the influence of a certain group of music clips. The ‘preferred fundamental’ is the one whose probability is the maximum under the effect of a certain music and has been coined with a new terminology “neural pitch”. The “neural pitch” of an EEG signal is actually the most basic and simplest feature of all other which have been used till date. The probability distribution curves obtained from the values of ZCR is essentially the void probability distribution calculated over time. The neural pitch analysis can thus be extended in the form of DFA/MFDFA analysis to study the fluctuation pattern of the void probability distribution. This will enable us to have an estimate of the fractal/multifractal scaling pattern (if any) of the probability distribution of voids. We sincerely believe that this feature has immense potential in categorization and classification of different EEG brain states. The new features in the domain of EEG signal processing viz. neural jitter, shimmer and pitch have been discussed at length in Chap. 10 of this book.

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